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Commutation technique for an exciton photocreated close to a metal

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Abstract. Recently, we have derived the changes in the absorption spectrum of an exciton when this exciton is photocreated close to a metal. The resolution of this problem – which has similarities with Fermi edge singularities – has been made possible by the introduction of "exciton diagrams" in which the exciton metal-electron vertex has been essentially guessed. The validity of this procedure relied on a dreadful calculation based on standard free electron and free hole diagrams, with the semiconductor-metal interaction included at second order only, and its guessed extention to higher orders. Using a commutation technique similar to the one we recently introduced to deal with interacting excitons, we are now able to *prove* that this exciton diagram procedure we proposed is indeed valid for this problem at any order in the interaction.

PACS. 71.10.Ca Electron gas, Fermi gas – 71.35.-y Excitons and related phenomena

Interactions with excitons have always been a tricky problem to handle properly. The interactions being in fact interactions with free electrons and free holes, one *a priori* has to crack the excitons into electrons and holes, in order to really know their effects. This leads to see the exciton as the sum of ladder diagrams [1] between one electron and one hole, with possibly, once in a while, an interaction of this electron or this hole with something else. Although fully safe, this approach becomes very fast dreadfully complicated, as can be seen from the simplest problem on interacting excitons studied in reference [2], namely an exciton photocreated close to a metallic "mirror". It is indeed the simplest problem on interacting excitons, in the sense that the photocreated electron and the metal electrons are discernable (being spatially separated) so that there is no Pauli exclusion between them. This Pauli exclusion, and the exchange processes associated to the indiscernability of the carriers, is an additional, but major, difficulty for interacting exciton problems. Rather recently, we have developed a "commutation technique" [3,4] which allows to cleanly identify contributions coming from Coulomb interaction *between excitons* and contributions coming from possible exchange *between carriers*. Using this commutation technique, we can derive the correlations between excitons at any order *exactly*. We have already been able to prove that the effective bosonic Hamiltonian for excitons quoted by everyone up to now cannot be correct: First, it is not even hermitian [3]; second, it misses purely Pauli terms [3]; third, and worse, the concept of effective Hamiltonian itself has to be given up [5] because, whatever the exciton-exciton part is, it cannot reproduce the exciton correlations correctly, due to the complexity of the exchange processes. If such an effective Hamiltonian were correct, exciton diagrams, with boson-exciton propagators and interaction vertices deduced from the interacting part of the Hamiltonian, would of course be fully valid. However, as such an effective Hamiltonian is incorrect, the validity of the exciton diagram procedure is actually not established at all in spite of a widely spread belief.

At the time we studied the problem of an exciton photocreated close to a metal and the changes in the exciton absorption spectrum induced by the semiconductor-metal interaction, we had not yet developed this commutation technique. This is why we safely used standard diagrams [6] with free electrons and free holes and Coulomb interactions between them. We were able to put the electron-metal and hole-metal interactions at second order only. At this order, we proved that the sum of all the seven complicated diagrams corresponding to these second order processes ends up with the same result as the one derived in an extremely simple way, by using intuitive "exciton diagrams": In these, the exciton propagator was taken to be

$$G_x(\omega;\nu,\mathbf{Q}) = \frac{1}{\omega - E_{\nu,\mathbf{Q}} + \mathrm{i}\eta}, \qquad (1)$$

where $E_{\nu,\mathbf{Q}} = \varepsilon_\nu + \mathcal{E}_\mathbf{Q}$ is the energy of the $(\nu,\mathbf{Q})$ exciton, ν being the relative motion state index and $\mathbf{Q}$ the center of

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mass momentum. The exciton-metal vertex was somehow *cooked in a reasonable way* from the bare electron-metal and hole-metal interactions.

Since there were no hope to calculate standard electron-hole diagrams with more than two electron-metal and hole-metal interactions, we assumed that the exciton diagram procedure, which looked physically quite reasonable, should hold at any order.

By studying this problem in the light of our commutation technique, we are now able to *prove* that this exciton diagram procedure is indeed fully correct in the studied case, at any order in the semiconductor-metal interaction.

Let us reconsider this problem from the beginning: A highly doped 2D quantum well is set at a distance d from an empty quantum well in which one exciton is photocreated. The metal Fermi sea reacts to the sudden appearance of the photocreated electron-hole, and its change, in turn, modifies the photon absorption. Of course, similarities with Fermi edge singularities [7–9] follow from this Fermi sea reaction. Note that in this problem we have one exciton only, so that we do not have to take into account any kind of exciton-exciton interactions.

The Hamiltonian of this semiconductor-metal coupled system reads $H = H_{\rm sc} + H_{\rm m} + W_{\rm sc-m}$, where $H_{\rm sc}$ is the semiconductor Hamiltonian and $H_{\rm m}$ is the metal Hamiltonian. The semiconductor-metal potential $W_{\rm sc-m}$ reads

$$W_{\rm sc-m} = \sum_{\mathbf{q}\neq\mathbf{0}} \sum_{\mathbf{k}} v(\mathbf{q}) \left(a^\dagger_{\mathbf{k}+\mathbf{q}} a_{\mathbf{k}} - b^\dagger_{\mathbf{k}+\mathbf{q}} b_{\mathbf{k}} \right) C_{-\mathbf{q}}, \qquad (2)$$

$$C_{\mathbf{q}} = \sum_{\mathbf{p}} c^\dagger_{\mathbf{p}+\mathbf{q}} c_{\mathbf{p}} \ , \qquad (3)$$

$a^\dagger_{\mathbf{k}}$, $b^\dagger_{\mathbf{k}}$ and $c^\dagger_{\mathbf{k}}$ being the semiconductor electron, semiconductor hole and metal electron creation operators respectively, while, for metal and semiconductor d apart [2], $v(\mathbf{q}) = \mathrm{e}^{-qd} 2\pi e^2/Sq$. Let us again stress that there is no way (and no hope) to rewrite this potential in terms of exciton operators, *i.e.*, linear combinations of $a^\dagger b^\dagger$ as defined in equation (5). This is why an interacting exciton *potential*, *i.e.*, an effective exciton Hamiltonian, has to result from approximations which are usually more physically reasonable than fully controlled.

The absorption of a photon $(\Omega, \mathbf{Q})$, given by the Fermi golden rule, is proportional to the imaginary part of the response function

$$S(\Omega, \mathbf{Q}) = \langle i | U \frac{1}{\Omega + \mathbb{E}_0 - H + \mathrm{i}\eta} U^\dagger | \mathrm{i} \rangle \ , \qquad (4)$$

where the initial state is $|i\rangle = |v\rangle \otimes |0\rangle$, with $|v\rangle$ being the semiconductor vacuum state and $|0\rangle$ the metal ground state, $(H_{\rm m} - \mathbb{E}_0)|0\rangle = 0$.

The excitons (*i.e.* all bound and extended one-pair eigenstates of the semiconductor Hamiltonian, $(H_{\rm sc} - E_{\nu,\mathbf{Q}}) B^\dagger_{\nu,\mathbf{Q}} |v\rangle = 0$) are related to the free pairs by

$$B^\dagger_{\nu,\mathbf{Q}} = \sum_{\mathbf{k}} \langle \mathbf{k} | x_\nu \rangle \, a^\dagger_{\mathbf{k}+\alpha_e \mathbf{Q}} \, b^\dagger_{-\mathbf{k}+\alpha_h \mathbf{Q}} \ , \qquad (5)$$

$$a^\dagger_{\mathbf{k}_e} \, b^\dagger_{\mathbf{k}_h} = \sum_{\nu} \langle x_\nu | \alpha_h \mathbf{k}_e - \alpha_e \mathbf{k}_h \rangle \, B^\dagger_{\nu, \mathbf{k}_e + \mathbf{k}_h} \ , \qquad (6)$$

where $\alpha_e = 1 - \alpha_h = m_e/(m_e + m_h)$, m_e and m_h being the electron and hole masses. Using equation (6), the semiconductor-photon interaction reads

$$U^\dagger = A \sum_{\mathbf{k}} a^\dagger_{\mathbf{k}+\mathbf{Q}} \, b^\dagger_{-\mathbf{k}} = A \sum_{\nu, \mathbf{k}'} \langle x_\nu | \mathbf{k}' \rangle \, B^\dagger_{\nu,\mathbf{Q}} = A \sum_{\nu} B^\dagger_{\nu,\mathbf{Q}} \langle x_\nu | \mathbf{r} = \mathbf{0} \rangle \ , \qquad (7)$$

(if we set the sample volume equal to 1). The response function thus appears as

$$S(\Omega, \mathbf{Q}) = A^2 \sum_{\nu,\nu'} \langle \mathbf{r} = \mathbf{0} | x_{\nu'} \rangle \, S_{\nu'\nu}(\Omega, \mathbf{Q}) \langle x_\nu | \mathbf{r} = \mathbf{0} \rangle, \qquad (8)$$

$$S_{\nu'\nu}(\Omega, \mathbf{Q}) = \langle i | B_{\nu',\mathbf{Q}} \frac{1}{a - H} B^\dagger_{\nu,\mathbf{Q}} | i \rangle \ , \quad a = \Omega + \mathbb{E}_0 + \mathrm{i}\eta. \qquad (9)$$

In order to calculate $S_{\nu'\nu}(\Omega, \mathbf{Q})$, we can note that

$$\left[H, B^\dagger_{\nu,\mathbf{Q}} \right] = \left[H_{\rm sc}, B^\dagger_{\nu,\mathbf{Q}} \right] + \left[W_{\rm sc-m}, B^\dagger_{\nu,\mathbf{Q}} \right] = (E_{\nu,\mathbf{Q}} B^\dagger_{\nu,\mathbf{Q}} + V^\dagger_{\nu,\mathbf{Q}}) + W^\dagger_{\nu,\mathbf{Q}}. \qquad (10)$$

The first commutator, calculated in reference [3], shows that $V^\dagger_{\nu,\mathbf{Q}}$ acts on semiconductor electron-hole pairs only so that $V^\dagger_{\nu,\mathbf{Q}} |v\rangle = 0$. Using equations (2, 5, 6), the second commutator gives

$$W^\dagger_{\nu,\mathbf{Q}} = \sum_{\mathbf{q}\neq\mathbf{0},\nu'} \hat{v}_{\nu'\nu}(\mathbf{q}) \, B^\dagger_{\nu',\mathbf{Q}+\mathbf{q}} \, C_{-\mathbf{q}}, \qquad (11)$$

$$\hat{v}_{\nu'\nu}(\mathbf{q}) = \langle x_{\nu'} | v(\mathbf{q}) (\mathrm{e}^{\mathrm{i}\alpha_h \mathbf{q}.\mathbf{r}} - \mathrm{e}^{-\mathrm{i}\alpha_e \mathbf{q}.\mathbf{r}}) | x_\nu \rangle = \langle x_{\nu'} | \hat{V}(\mathbf{q}) | x_\nu \rangle \cdot \qquad (12)$$

$W^\dagger_{\nu,\mathbf{Q}}$ physically corresponds to excite one exciton from a $(\nu, \mathbf{Q})$ state to a $(\nu', \mathbf{Q} + \mathbf{q})$ state, whereas the metal has one of its electrons excited from $\mathbf{p}$ to $\mathbf{p} - \mathbf{q}$. Let us however stress that $W^\dagger_{\nu,\mathbf{Q}}$ is *not* an exciton-Fermi sea interacting potential. Such a potential, which would read as $B^\dagger_{\nu',\mathbf{Q}+\mathbf{q}} a^\dagger_{\mathbf{p}-\mathbf{q}} a_{\mathbf{p}} B_{\nu,\mathbf{Q}}$, does not exist as previously discussed. It is of importance to note that our commutation technique avoids such a potential while keeping the exact Coulomb interaction between the exciton and the metal electrons.

It is easy to check that equation (10) leads to

$$\frac{1}{a - H} B^\dagger_{\nu,\mathbf{Q}} = B^\dagger_{\nu,\mathbf{Q}} \frac{1}{a - H - E_{\nu,\mathbf{Q}}} + \frac{1}{a - H} (V^\dagger_{\nu,\mathbf{Q}} + W^\dagger_{\nu,\mathbf{Q}}) \frac{1}{a - H - E_{\nu,\mathbf{Q}}} \cdot \qquad (13)$$

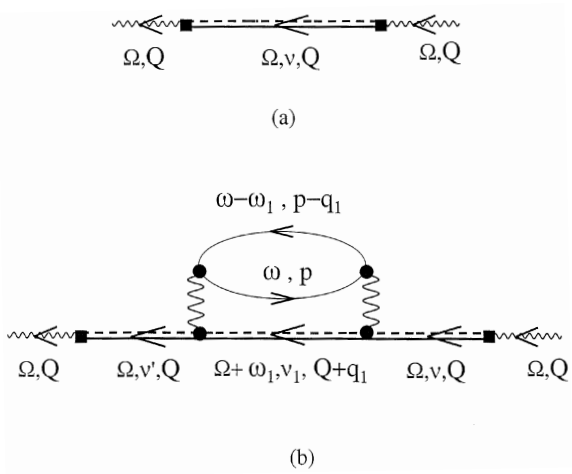

Fig. 1. Response function in terms of exciton diagrams, at zero order (a) and at second order (b) in the semiconductor-metal interaction. To the $(\Omega;\nu,\mathbf{Q})$ exciton, we associate the exciton propagator $G_x(\Omega;\nu,\mathbf{Q})$ given in equation (1), and to the scattering of a $(\Omega;\nu,\mathbf{Q})$ exciton-$(\omega,\mathbf{p})$ metal electron into a $(\Omega+\omega_1;\nu_1,\mathbf{Q}+\mathbf{q}_1)$ exciton-$(\omega-\omega_1,\mathbf{p}-\mathbf{q}_1)$ metal electron, we associate the exciton-metal vertex $\hat{V}_{\nu_1\nu}(\mathbf{q}_1)$ given in equation (12).

As $V^\dagger_{\nu,\mathbf{Q}}|v\rangle = 0$, while $W^\dagger_{\nu,\mathbf{Q}}|v\rangle$ writes in terms of $B^\dagger_{\nu',\mathbf{Q}'}$, the iteration of the above equation allows to generate the expansion of $S_{\nu'\nu}(\Omega,\mathbf{Q})$ in the exciton-metal interaction:

$$S_{\nu'\nu}(\Omega,\mathbf{Q}) = \sum_{n=0}^{\infty} S^{(n)}_{\nu'\nu}(\Omega,\mathbf{Q})\ . \tag{14}$$

The zero order term simply comes from the first term of equation (13). It reads

$$\begin{aligned} S^{(0)}_{\nu'\nu}(\Omega,\mathbf{Q}) &= \langle i|B_{\nu',\mathbf{Q}}\, B^\dagger_{\nu,\mathbf{Q}}\, \frac{1}{a-H-E_{\nu,\mathbf{Q}}}\,|i\rangle \\ &= \delta_{\nu',\nu} G_x(\Omega;\nu,\mathbf{Q}), \end{aligned} \tag{15}$$

and thus corresponds to the exciton diagram of Figure 1a. The first order term appears as

$$\begin{aligned} S^{(1)}_{\nu'\nu}(\Omega,\mathbf{Q}) = \sum_{\mathbf{q}_1\neq 0,\nu_1} \langle i|B_{\nu',\mathbf{Q}} B^\dagger_{\nu_1,\mathbf{Q}+\mathbf{q}_1} \frac{1}{a-H-E_{\nu_1,\mathbf{Q}+\mathbf{q}_1}} \\ \times \hat{W}_{-\mathbf{q}_1;\nu_1\nu}|i\rangle\ G_x(\Omega;\nu,\mathbf{Q}), \end{aligned} \tag{16}$$

where we have set $\hat{W}_{-\mathbf{q};\nu'\nu} = \hat{v}_{\nu'\nu}(\mathbf{q})C_{-\mathbf{q}}$. As $\langle v|B_{\nu',\mathbf{Q}}B^\dagger_{\nu_1,\mathbf{Q}+\mathbf{q}_1}|v\rangle = \delta_{\nu',\nu_1}\delta_{\mathbf{q}_1,\mathbf{0}}$, this first order term is equal to zero.

The second order term, shown in Figure 1b, reads

$$\begin{aligned} S^{(2)}_{\nu'\nu}(\Omega,\mathbf{Q}) = \\ \sum_{\mathbf{q}_2\neq\mathbf{0},\nu_2} \sum_{\mathbf{q}_1\neq\mathbf{0},\nu_1} \langle i|B_{\nu',\mathbf{Q}} B^\dagger_{\nu_2,\mathbf{Q}+\mathbf{q}_1+\mathbf{q}_2} \frac{1}{a-H-E_{\nu_2,\mathbf{Q}+\mathbf{q}_1+\mathbf{q}_2}} \\ \times \hat{W}_{-\mathbf{q}_2;\nu_2\nu_1} \frac{1}{a-H-E_{\nu_1,\mathbf{Q}+\mathbf{q}_1}} \hat{W}_{-\mathbf{q}_1;\nu_1\nu}|i\rangle\ G_x(\Omega;\nu,\mathbf{Q}). \end{aligned} \tag{17}$$

The above matrix element can be split into a semiconductor part and a metal part. The first one imposes $\nu_2 = \nu'$ and $\mathbf{q}_1+\mathbf{q}_2 = 0$, so that equation (17) becomes

$$S^{(2)}_{\nu'\nu}(\Omega,\mathbf{Q}) = G_x(\Omega;\nu',\mathbf{Q})\ T^{(2)}_{\nu'\nu}(\Omega,\mathbf{Q})\, G_x(\Omega;\nu,\mathbf{Q}), \tag{18}$$

$$\begin{aligned} T^{(2)}_{\nu'\nu}(\Omega,\mathbf{Q}) = \sum_{\mathbf{q}_1\neq 0,\nu_1} \hat{v}_{\nu'\nu_1}(-\mathbf{q}_1) \\ \times \langle 0|C_{\mathbf{q}_1} \frac{1}{a-H_{\rm m}-E_{\nu_1,\mathbf{Q}+\mathbf{q}_1}} C_{-\mathbf{q}_1}|0\rangle \hat{v}_{\nu_1\nu}(\mathbf{q}_1), \end{aligned} \tag{19}$$

with ν_1 being the intermediate exciton state. Following reference [2], we neglect Coulomb interaction between metal electrons for simplicity, as it does not play any role in the excitonic singularity we are studying – which only comes from the sudden appearance of the photocreated exciton. We then get $H_{\rm m}\, c^\dagger_{\mathbf{p}-\mathbf{q}}c_{\mathbf{p}}|0\rangle = (\mathbb{E}_0+\epsilon_{\mathbf{p}-\mathbf{q}}-\epsilon_{\mathbf{p}})\, c^\dagger_{\mathbf{p}-\mathbf{q}}c_{\mathbf{p}}|0\rangle$, $\epsilon_{\mathbf{p}}$ being the metal-electron energy. The matrix element of equation (19) is thus equal to

$$\sum_{|\mathbf{p}|<k_{\rm F}<|\mathbf{p}-\mathbf{q}_1|} \frac{1}{\Omega-(E_{\nu_1,\mathbf{Q}+\mathbf{q}_1}+\epsilon_{\mathbf{p}-\mathbf{q}_1}-\epsilon_{\mathbf{p}})+{\rm i}\eta}\cdot \tag{20}$$

We can split it into contributions from the exciton $(\nu_1,\mathbf{Q}+\mathbf{q}_1)$, the metal electron $(\mathbf{p}-\mathbf{q}_1)$ and the metal hole $(\mathbf{p})$ by using the standard trick,

$$\begin{aligned} \frac{1}{\Omega-a-b+{\rm i}\eta} = \\ \int \frac{{\rm i}d\omega}{2\pi} \left(\frac{1}{\omega+\Omega-a+{\rm i}\eta}\right)\left(\frac{1}{-\omega-b+{\rm i}\eta}\right)\cdot \end{aligned} \tag{21}$$

Equation (20) then reads

$$\begin{aligned} \int \frac{{\rm i}d\omega_1}{2\pi}\, G_x(\omega_1+\Omega;\nu_1,\mathbf{Q}+\mathbf{q}_1) \\ \times \left[-\sum_{\mathbf{p}} \int \frac{{\rm i}d\omega}{2\pi}\, g(\omega,\mathbf{p})\, g(\omega-\omega_1,\mathbf{p}-\mathbf{q}_1)\right], \end{aligned} \tag{22}$$

where $g(\omega,\mathbf{p}) = (\omega-\epsilon_{\mathbf{p}}+{\rm i}\eta\,{\rm sign}(\epsilon_{\mathbf{p}}-\mu))^{-1}$ is the usual metal-electron Green's function, while $G_x(\omega;\nu,\mathbf{Q})$ is the "exciton Green's function" given in equation (1). This leads to rewrite $T^{(2)}_{\nu'\nu}(\Omega,\mathbf{Q})$ as

$$\begin{aligned} T^{(2)}_{\nu'\nu}(\Omega,\mathbf{Q}) = \sum_{\mathbf{q}_1\neq 0,\nu_1} \int \frac{{\rm i}d\omega_1}{2\pi}\, B(\omega_1,\mathbf{q}_1) \\ \times \left[\hat{v}_{\nu'\nu_1}(-\mathbf{q}_1)\, G_x(\Omega+\omega_1;\nu_1,\mathbf{Q}+\mathbf{q}_1)\, \hat{v}_{\nu_1\nu}(\mathbf{q}_1)\right], \end{aligned} \tag{23}$$

$B(\omega_1,\mathbf{q}_1)$ being the standard "bubble" contribution as given by the bracket of equation (22). This response function second order term, as well as the zero order term given in equation (15), correspond to the exciton diagrams shown in Figure 1, with the exciton-metal vertex being $\hat{v}_{\nu'\nu}(\mathbf{q})$. Note that the intermediate exciton

$(\nu_1, \mathbf{Q}+\mathbf{q_1})$ now appears clearly through its propagator $G_x(\Omega+\omega_1;\nu_1,\mathbf{Q}+\mathbf{q_1})$.

More generally, the nth order term of $S_{\nu'\nu}(\Omega,\mathbf{Q})$ is given by

$$S^{(n)}_{\nu'\nu}(\Omega,\mathbf{Q}) = G_x(\Omega;\nu',\mathbf{Q})\,G_x(\Omega;\nu,\mathbf{Q})\left[\sum_{\mathbf{q}_{n-1}\neq\mathbf{0},\nu_{n-1}}\cdots\right.$$
$$\left.\sum_{\mathbf{q}_1\neq\mathbf{0},\nu_1}\langle 0|\hat{W}_{\mathbf{q}_{n-1}+\cdots+\mathbf{q}_1;\nu'\nu_{n-1}}\,M_{n-1}\,M_{n-2}\cdots M_1|0\rangle\right],$$
$$M_m = \frac{1}{a-H_{\rm m}-E_{\nu_m,\mathbf{Q}+\mathbf{q}_m+\cdots+\mathbf{q}_1}}\,\hat{W}_{-\mathbf{q}_m;\nu_m\nu_{m-1}}(\nu_0\equiv\nu). \quad (24)$$

The bracket corresponds to all the possible ways to start with a $(\nu,\mathbf{Q})$ exciton, to excite this unique exciton into various $(\nu'',\mathbf{Q}+\mathbf{q}'')$ states while shaking up the metal Fermi sea by $(-\mathbf{q}'')$ and to end with a $(\nu',\mathbf{Q})$ exciton. As an example, the 4th order terms are shown in Figure 2. They are basically of two types: The first term (Fig. 2a) corresponds to excite and recombine one electron-hole pair in the metal Fermi sea, twice. Its contribution to $S^{(4)}_{\nu'\nu}(\Omega,\mathbf{Q})$ is given by

$$G_x(\Omega;\nu',\mathbf{Q})\left[\sum_{\nu_1}T^{(2)}_{\nu'\nu_1}(\Omega,\mathbf{Q})\,G_x(\Omega;\nu_1,\mathbf{Q})\,T^{(2)}_{\nu_1\nu}(\Omega,\mathbf{Q})\right]$$
$$\times\,G_x(\Omega;\nu,\mathbf{Q}). \quad (25)$$

The other terms of Figure 2 can be formally written as

$$G_x(\Omega;\nu',\mathbf{Q})\,T^{(4)}_{\nu'\nu}(\Omega,\mathbf{Q})\,G_x(\Omega;\nu,\mathbf{Q}), \quad (26)$$

where $T^{(4)}_{\nu'\nu}(\Omega,\mathbf{Q})$ corresponds to the transfer of the $(\nu,\mathbf{Q})$ exciton into the $(\nu',\mathbf{Q})$ state associated to all possible *connected* excitation processes of the metal Fermi sea with 4 semiconductor-metal interactions.

This shows that the sum of all contributions to $S_{\nu'\nu}(\Omega,\mathbf{Q})$ reads

$$S_{\nu'\nu}(\Omega,\mathbf{Q}) = \delta_{\nu',\nu}\,G_x(\Omega;\nu,\mathbf{Q}) + G_x(\Omega;\nu',\mathbf{Q})\left[T_{\nu'\nu}(\Omega,\mathbf{Q})\right.$$
$$\left.+\sum_{\nu_1}T_{\nu'\nu_1}(\Omega,\mathbf{Q})G_x(\Omega;\nu_1,\mathbf{Q})T_{\nu_1\nu}(\Omega,\mathbf{Q})+\cdots\right]$$
$$\times\,G_x(\Omega;\nu,\mathbf{Q}), \quad (27)$$

where $T_{\nu'\nu}(\Omega,\mathbf{Q})$ corresponds to the transfer of a $(\nu,\mathbf{Q})$ exciton into a $(\nu',\mathbf{Q})$ state associated to the sum of all possible *connected* excitation processes of the metal Fermi sea with two or more semiconductor-metal interactions. This expansion of $S_{\nu'\nu}(\Omega,\mathbf{Q})$ is shown in Figure 3. It corresponds to the expansion of the integral equation shown in Figure 3.

It is in fact possible to rewrite $S_{\nu'\nu}(\Omega,\mathbf{Q})$, as well as $S(\Omega,\mathbf{Q})$, in a quite compact form: For that, we first

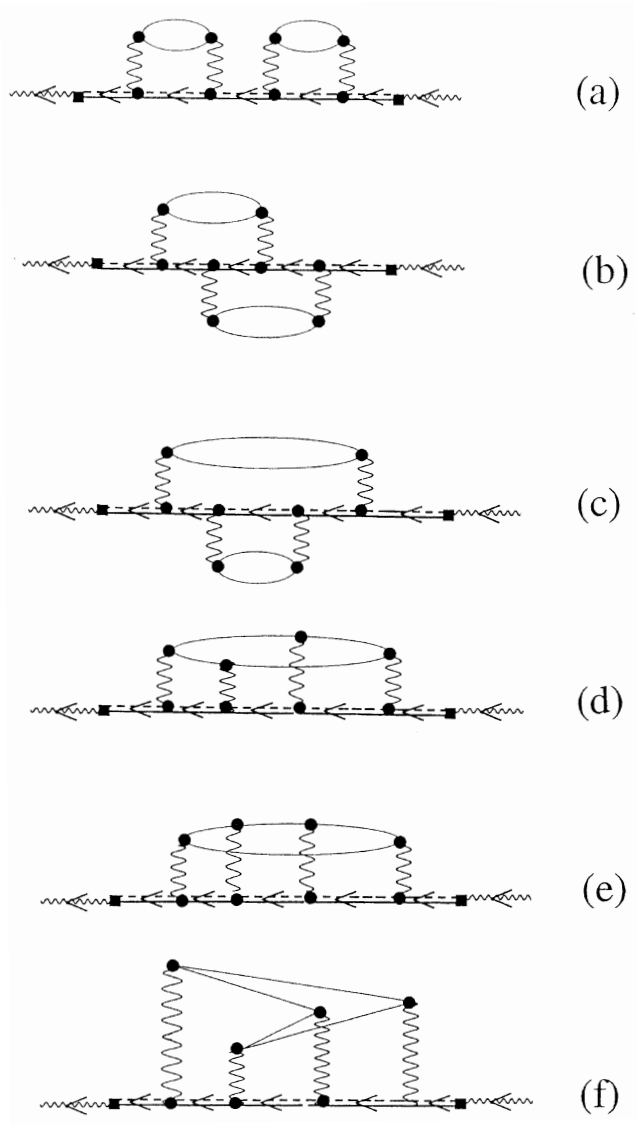

Fig. 2. Exciton diagrams for the response function at 4th order in the semiconductor-metal interaction.

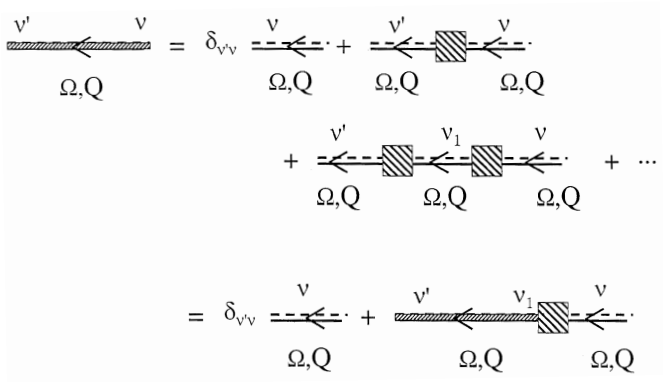

Fig. 3. Diagrammatic representation of the integral equation verified by $S_{\nu'\nu}(\Omega,\mathbf{Q})$ as given in equation (27).

rewrite the exciton propagator as

$$G_x(\Omega;\nu,\mathbf{Q}) = \langle x_\nu | \frac{1}{\Omega - h_x - \mathcal{E}_{\mathbf{Q}} + \mathrm{i}\eta} | x_\nu \rangle, \qquad (28)$$

where h_x is the exciton relative motion Hamiltonian, $(h_x - \varepsilon_\nu)|x_\nu\rangle = 0$. By noting that the second order transfer, given in equation (23), also reads $T^{(2)}_{\nu'\nu}(\Omega,\mathbf{Q}) = \langle x_{\nu'}|T^{(2)}(\Omega,\mathbf{Q})|x_\nu\rangle$ with

$$T^{(2)}(\Omega,\mathbf{Q}) = \sum_{\mathbf{q}\neq\mathbf{0}} \int \frac{\mathrm{id}\omega}{2\pi}\, B(\omega,\mathbf{q})\, \hat{V}(-\mathbf{q}) \times \frac{1}{\Omega + \omega - h_x - \mathcal{E}_{\mathbf{Q}+\mathbf{q}} + \mathrm{i}\eta}\, \hat{V}(\mathbf{q}) \ , \qquad (29)$$

we can, in a similar way, rewrite the higher order transfers as $T_{\nu'\nu}(\Omega,\mathbf{Q}) = \langle x_{\nu'}|T(\Omega,\mathbf{Q})|x_\nu\rangle$. Since $\delta_{\nu',\nu} = \langle x_{\nu'}|x_\nu\rangle$, equation (27) is nothing but the expansion of

$$S_{\nu'\nu}(\Omega,\mathbf{Q}) = \langle x_{\nu'}| \frac{1}{\Omega - h_x - T(\Omega,\mathbf{Q}) - \mathcal{E}_{\mathbf{Q}} + \mathrm{i}\eta} |x_\nu\rangle, \qquad (30)$$

so that the response function $S(\Omega,\mathbf{Q})$ given in equation (8) takes the quite compact form,

$$S(\Omega,\mathbf{Q}) = A^2 \langle \mathbf{r}=\mathbf{0}| \frac{1}{\Omega - h_x - T(\Omega,\mathbf{Q}) - \mathcal{E}_{\mathbf{Q}} + \mathrm{i}\eta} |\mathbf{r}=\mathbf{0}\rangle \cdot \qquad (31)$$

The above equation is exactly the equation (9) of reference [1]. The explicit form of this response function was then obtained in terms of the right and left eigenstates $|\hat{x}_\nu\rangle$ and $|\hat{\tilde{x}}_\nu\rangle$ of the non hermitian "Hamiltonian" $h_x + T(\Omega,\mathbf{Q})$. As its eigenvalues are complex, the exciton absorption lines in the presence of a 2D metal have now tails.

In conclusion, our commutation technique allows to prove in a quite transparent way that the problem of the exciton absorption spectrum changes induced by the presence of a distant metal, can indeed be solved within exciton diagrams at any order in the semiconductor-metal interaction. These diagrams visualize the fact that a $(\nu,\mathbf{Q})$ exciton is created by a $\mathbf{Q}$ photon. This $(\nu,\mathbf{Q})$ exciton scatters to a $(\nu_1,\mathbf{Q}+\mathbf{q}_1)$ state and then to a $(\nu_2,\mathbf{Q}+\mathbf{q}_1+\mathbf{q}_2)$ state and so on ... At each $\mathbf{q}_i$ scattering, a $(-\mathbf{q}_i)$ metal electron-metal hole pair is excited. The photocreated exciton must end all these scatterings in a $(\nu',\mathbf{Q})$ state in order to possibly recombine into a $\mathbf{Q}$ photon. From a technical point of view, these exciton diagrams are such that:

– to each $(\nu',\mathbf{Q}')$ exciton we associate the propagator $G_x(\omega;\nu',\mathbf{Q}')$ given in equation (1);

– to each scattering of a $(\nu,\mathbf{Q};\mathbf{p})$ exciton-metal-electron state into a $(\nu',\mathbf{Q}+\mathbf{q};\mathbf{p}-\mathbf{q})$ state we associate the exciton-metal vertex $\hat{V}_{\nu'\nu}(\mathbf{q})$ given in equation (12);

– as usual for diagrams, we conserve ω and $\mathbf{q}$ at each vertex.

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